Math 422, Homework 1

[Your name]
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1.1.1(b)(d)

"Suppose |A| = m and |B| = n, where m and n are finite, and let f : A → B be a function from A to B.

(b) What can you say about m and n if f is one-to-one? if f is onto? if f is a bijection.

(d) How many possible one-to-one functions are there from A to B? How many possible functions are there from A to B? How many relations are there from A to B?"

Proof. □

1.2.6

"Prove that if G is a set with an associative binary operation * such that both of the equations a * x = b and y * a = b have unique solutions whenever a and b are in G, then G is a group."

Proof. □

1.2.8

"Prove that if G is a group such that g^2 = e for all g ∈ G, then G is abelian."

Proof. □

1.3.4

"Let p_1, p_2, . . . , p_k be the first k prime numbers, and set N_k = (p_1 ∙ p_2 . . . p_k) + 1. It is easy to see that N_k is prime for 1 ≤ k ≤ 4. Is N_k always prime?"

Proof. □
1.3.8

“Show that if \((a, b) = (a, c) = 1\), then \((a, bc) = 1\).”

Proof.

1.3.11(c)

“Suppose that \((a, b) = 1\). Show that \((a + b, a^2 + b^2)\) is equal to 1 or 2.”

Proof.

1.3.16(a)

“Let \(F_0 , F_1 , F_2 , \ldots\) be the Fibonacci Sequence. In particular, \(F_0 = 0\), \(F_1 = 1\),
and for \(n \geq 2\) \(F_n = F_{n-1} + F_{n-2}\). Show that any pair of consecutive Fibonacci
numbers are relatively prime.”

Proof.

[Not from book.]

“Find the \(\text{gcd}\) of and Bezout’s coefficients for 15147 and 891800.”

Proof.

1.4.4

“Let \(n\) be a positive integer. Show that the relation on the integers \(\mathbb{Z}\) defined
by \(a \sim b \leftrightarrow n|(b − a)\) is an equivalence relation. (Theorem 1.4.3).”

Proof.